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THIN-LAYER FLOW OF A TWO-PHASE MEDIUM OVER THE SURFACE OF A CENTRIFUGAL MIXER, TAKING ACCOUNT OF RHEOLOGICAL FACTORS

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On the basis of a hydrodynamic model of multivelocity interpenetrating continua, the flow of miscible materials over the working surface of centrifugal mixers is investigated.

The mixing of highly disperse materials with viscous liquids is widely used in various branches of industry. Success in the development and commercial introduction of mixers that ensure a high quality of distribution of disperse materials in viscous liquids with sufficient productivity in continuous operation and allow the difficulties associated with the specific properties and treatment conditions of these materials to be overcome is only possible in the presence of a mathematical description of the processes occurring in them on mixing. One apparatus satisfying these requirements is the multicascade centrifugal mixer. A possible form of this type of mixer is shown in Fig. 1. In the present work, the flow of miscible materials over the working surface of the mixer is described, allowing the optimal constructional elements of the apparatus to be determined.

In the operation of a rotational mixer, highly disperse liquid and solid components are fed to the center of the rotating element of the first stage of the rotor and, in layers, pure liquid, the mixture forming, and the solid component flow over the surface of the conical ring. As a result of their combined motion under the action of the centrifugal force $\bar{F}$, collective deposition of the solid material in the liquid occurs, and then all this mass is dispersed by the edge of the rotating element (Fig. 1). The processes occurring in the subsequent stages of the rotor, which are analogous in construction, are flow of the two-phase medium and dispersion, as a result of which the final redistribution of the components is accomplished.

The motion of each layer of material over the surface of the rotating rotor is described by the equations of continuum mechanics; each layer corresponds to a particular rheological equation of state. The flow of pure liquid is described by the Navier—Stokes equation, and the mixture constitutes a two-phase medium, the flow of which may be described using the results of [1, 2]. The motion of the highly disperse material may be regarded as motion of some continuous medium according to its rheological equation of state. In this case, however, the general problem is greatly complicated; therefore, it is assumed that the solid material moves in the longitudinal direction at some mean velocity $V_l[\delta_1(l)]$, and that there is no relative motion at the interface between the mixture and the solid phase. These assumptions are completely justified for the given case of flow.

Let $\delta_0(l)$, $\delta_1(l)$, $\delta_2(l)$ be the thickness of the pure liquid layer, the pure liquid layer and the layer of mixture forming, taken together, and the thickness of the whole layer, respectively. Consider the flow of miscible materials in the orthogonal coordinate system x^1, x^2, x^3 fixed with respect to the rotor (Fig. 1), under the following assumptions, which are valid for the given type of flow: the flow is axisymmetric and steady; the film thickness is considerably less than the corresponding radius of the conical rotor ring, i.e., $\delta_2/R = \varepsilon << 1$;

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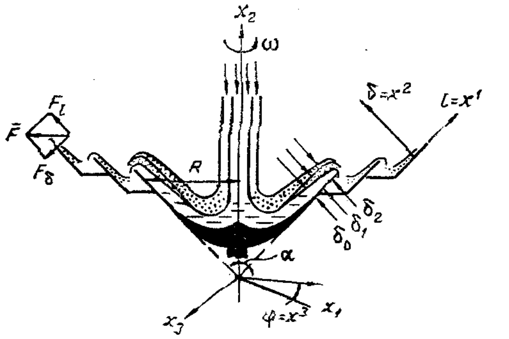

Fig. 1. Flow diagram of miscible materials at the working surface of a rotational mixer.

the Coriolis forces are neglected in comparison with the centrifugal forces $F_l = \omega^2 R\sin\alpha$ and $F_\delta = -\omega^2 R\cos\alpha$.

Then the equations describing the motion of the liquid and the two-phase mixture, taking account of these assumptions, after evaluating the significance of the terms, according to [3, 4], are written in the form

$$\frac{\partial(RV_{10x^1})}{\partial x^1}+\frac{\partial(RV_{10x^2})}{\partial x^2}=0, \tag{1}$$

$$\frac{\partial}{\partial x^2}\left(\mu_1\frac{\partial V_{10x^1}}{\partial x^2}\right)+\rho_1^0F_{x^1}=0, \tag{2}$$

$$-\frac{\partial P_{10}}{\partial x^2}+\rho_1^0F_{x^2}=0, \tag{3}$$

$$\frac{\partial(\alpha_1RV_{1x^1})}{\partial x^1}+\frac{\partial(\alpha_1RV_{1x^2})}{\partial x^2}=0, \tag{4}$$

$$\frac{\partial}{\partial x^2}\left(\mu(\alpha_2)\frac{\partial V_{1x^1}}{\partial x^2}\right)-f(\alpha_2)(V_{1x^1}-V_{2x^1})+\rho_1F_{x^1}=0, \tag{5}$$

$$-\alpha_1\frac{\partial P}{\partial x^2}-f(\alpha_2)(V_{1x^1}-V_{2x^2})+\rho_1F_{x^2}=0, \tag{6}$$

$$\frac{\partial(\alpha_2RV_{2x^1})}{\partial x_1}+\frac{\partial(\alpha_2RV_{2x^2})}{\partial x^2}=0, \tag{7}$$

$$f(\alpha_2)(V_{1x^1}-V_{2x^1})+\rho_2F_{x^1}=0, \tag{8}$$

$$-\alpha_2\frac{\partial P}{\partial x^2}+f(\alpha_2)(V_{1x^2}-V_{2x^2})+\rho_2F_{x^2}=0. \tag{9}$$

In solving the problem of the motion of a mass over the mixer surface, the determining values are the velocities of mixture motion along the rotor generatrix V_{10x^1}, V_{1x^1}, V_{2x^1}, the rate of deposition of solid particles in the centrifugal field V_{2x^2}, the transverse velocities V_{10x^2}, V_{1x^2}, and the layer thicknesses δ_1, δ_0, δ_2. If Eqs. (5) and (8) are added and the pressure is eliminated from Eqs. (6) and (9), the system of equations obtained is

$$\frac{\partial}{\partial x^1}(RV_{10x^1})+\frac{\partial}{\partial x^2}(RV_{10x^2})=0, \tag{10}$$

$$\frac{\partial}{\partial x^2}\left(\mu_1\frac{\partial V_{10x^1}}{\partial x^2}\right)+\rho_1^0F_{x^1}=0, \tag{11}$$

$$\frac{\partial}{\partial x^1}(\alpha_1RV_{1x^1})+\frac{\partial}{\partial x^2}(\alpha_1RV_{1x^2})=0, \tag{12}$$

$$\frac{\partial}{\partial x^2}\left(\mu(\alpha_2)\frac{\partial V_{1x^1}}{\partial x^2}\right)+(\rho_1+\rho_2)F_{x^1}=0, \tag{13}$$

$$f(\alpha_2)(V_{1x^1}-V_{2x^1})+\rho_2 F_{x^1}=0, \tag{14}$$

$$f(\alpha_2)(V_{1x^2}-V_{2x^2})-\alpha_1\alpha_2(\rho_1^0-\rho_2^0)F_{x^2}=0, \tag{15}$$

which are sufficient for the determination of the velocity fields that are of interest here. The boundary conditions for the system in Eqs. (10)-(15) take the form:

$$\text{when } x^2=0\ \ V_{10x^1}=V_{10x^2}=0, \tag{16a}$$

$$\text{when } x^2=\delta_1\ \ V_{10x^1}=V_{1x^1},\ V_{10x^2}=\alpha_1 V_{1x^2},\ \mu_1\frac{\partial V_{10x^1}}{\partial x^2}=\mu(\alpha_2)\frac{\partial V_{1x^1}}{\partial x^2}, \tag{16b}$$

$$\text{when } x^2=\delta_2\ \ \frac{\partial V_{1x^1}}{\partial x^2}=0. \tag{16c}$$

As a result of solving Eqs. (10)-(15) with the boundary conditions in Eq. (16), the following dependences are obtained:

$$V_{10l}=-\frac{\rho_1^0 F_l\delta^2}{2\mu_1}+\frac{C_1(l)\delta}{\mu_1}, \tag{17}$$

$$V_{10\delta}=\frac{(\sin\alpha F_l+RF_l')\rho_1^0\delta^3}{6\mu_1 R}-\frac{(\sin\alpha C_1+RC_1')}{2\mu_1 R}\delta^2, \tag{18}$$

$$V_{1l}=-\frac{\rho F_l\delta^2}{2\mu(\alpha_2)}+\frac{\alpha_2\Delta\rho F_l\delta_0\delta}{\mu(\alpha_2)}+\frac{C_1\delta}{\mu(\alpha_2)}+C_2(l), \tag{19}$$

$$V_{1\delta}=\frac{(\sin\alpha F_l+RF_l')}{6R}\left[\frac{\rho(\delta^3-\alpha_1\delta_0^3)}{\mu(\alpha_2)}+\frac{\rho_1^0\delta_0^3}{\mu_1}\right]-\frac{\alpha_2\Delta\rho(\sin\alpha F_l\delta_0+RF_l\delta_0'+RF_l'\delta_0)}{2\mu(\alpha_2)R}(\delta^2-\alpha_1\delta_0^2)-$$

$$-\frac{(\sin\alpha C_1+RC_1')}{2R}\left[\frac{(\delta^2-\alpha_1\delta_0^2)}{\mu(\alpha_2)}+\frac{\delta_0^2}{\mu_1}\right]-\frac{(\sin\alpha C_2+RC_2')}{R}(\delta-\alpha_1\delta_0), \tag{20}$$

$$V_{2l}=V_{1l}+\frac{\rho_2 F_l}{f(\alpha_2)}, \tag{21}$$

$$V_{2\delta}=V_{1\delta}+\frac{\alpha_1\alpha_2\Delta\rho F_\delta}{f(\alpha_2)}, \tag{22}$$

$$V_l(\delta_1)=V_{1l}(l,\ \delta_1), \tag{23}$$

where

$$C_1(l)=\rho F_l\delta_1-\alpha_2\Delta\rho F_l\delta_0;\ C_2(l)=\left(\frac{\rho-2\alpha_2\Delta\rho}{\mu(\alpha_2)}-\frac{\rho_1^0}{\mu_1}\right)\frac{F_l\delta_0^2}{2}+\delta_0 C_1(l)\left(\frac{1}{\mu_1}-\frac{1}{\mu(\alpha_2)}\right);$$

$\Delta\rho=\rho_2^0-\rho_1^0$; $\rho=\rho_1+\rho_2$; a prime denotes the derivative with respect to l.

To determine the unknowns δ_0, δ_1, δ_2, use is made of the conditions of constant flow rate of the solid and liquid components q_1 and q_2 and the mechanism of variation in the thickness δ_0, which may be written in the form

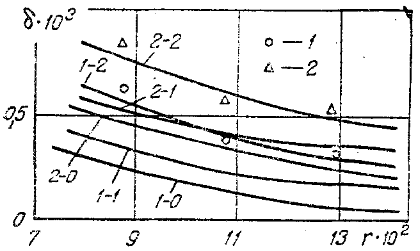

Fig. 2. Comparison of experimental and theoretical data on the layer-film thickness δ_2, and the theoretical values of δ_1, δ_0: 1) the system glycerin + KCl: $q_1 = 28\cdot10^{-3}$ kg/sec, $q_2 = 11\cdot10^{-3}$ kg/sec, $\omega = 200$ sec^{-1}; 2) glycerin + Al_2O_3: $q_1 = 28\cdot10^{-3}$ kg/sec, $q_2 = 22\cdot10^{-3}$ kg/sec, $\omega = 100$ sec^{-1}, where i–2, i–1, i–0 refer to δ_2, δ_1, δ_0, respectively, for the i-th system (i = 1, 2). δ, r, m.

$$\int_0^{\delta_0} 2\pi r \rho_1^0 V_{10l} d\delta + \int_{\delta_0}^{\delta_1} 2\pi r \alpha_1 \rho_1^0 V_{1l} d\delta = q_1, \tag{24}$$

$$\int_{\delta_0}^{\delta_1} 2\pi r \alpha_2 \rho_2^0 V_{2l} d\delta + 2\pi r \rho_{20} [V_l(\delta_1)](\delta_2 - \delta_1) = q_2, \tag{25}$$

$$\frac{d\delta_0}{dl} = \frac{V_{1\delta}(l,\ \delta_0)}{V_{1l}(l,\ \delta_0)} + \frac{V_{2\delta}(l,\ \delta_0) - V_{1\delta}(l,\ \delta_0)}{V_{2l}(l,\ \delta_0)}, \tag{26}$$

where $r = R - \delta\cos\alpha$; ρ_{20} is the bulk density of the solid phase.

The Runge–Kutta method is used to solve Eqs. (24) and (26) simultaneously, and δ_2 is determined from Eq. (25). Comparison of the theoretical and experimental data shows good agreement (Fig. 2). The results of the calculations shown by the continuous lines 1–j and 2–j correspond to the experimental values of δ_2 (j = 0, 1, 2). Solving this hydrodynamic problem allows the optimal length of the generatrix of the rotor first stage necessary for obtaining a high-quality mixture to be determined from one of the conditions: 1) $\delta_0 = 0$, 2) $\delta_2 - \delta_1 = 0$, 3) δ_0, $\delta_1 - \delta_0$, $\delta_2 - \delta_1$ are of the same order of magnitude as the dimensions of solid-particle agglomerations. Calculations by conditions 1) - 3) show that mixing of highly disperse materials with viscous liquids is expediently undertaken in multicascade mixers with an elongated first stage, the length of its generatrix being determined from condition 3). The number of mixer cascades necessary to obtain quality mixing may be determined from a mathematical model of the kinetics of the continuous mixing process [5]. For a broad class of materials, it is sufficient to use three- or four-cascade mixers.

As shown by the results of numerical calculations, the rheological properties of the carrier liquid and the middle layer of mixture have a strong influence on the flow conditions and hence on the optimal mixer dimensions. For example, when $q_1 = 28\cdot10^{-3}$ kg/sec, $q_2 = 31\cdot10^{-3}$ kg/sec, $\rho_1^0 = 1260$ kg/m³, $\rho_2^0 = 3960$ kg/m³, $\alpha_2 = 0.26$, $\omega = 200$ sec^{-1}, $a = 150$ µm, and at values of the carrier-phase viscosity $\mu_1 = 0.14$, 1.4, and 10 sec·N/m², the optimal mixer dimension determined from condition 3) is $R_C = 0.04$, 0.107, and 0.282 m, respectively. The longitudinal velocity averaged over the layer thicknesses is $V_{10l}(R_C) = 0.29$, $V_{1l}(R_C) = 0.44$; $V_{10l}(R_C) = 0.098$, $V_{1l}(R_C) = 0.156$; and $V_{10l}(R_C) = 0.036$ m/sec, $V_{1l}(R_C) = 0.958$ m/sec, respectively. All the flow characteristics are also influenced by the particle size and their concentration in the mixture α_2.

The motion (flow) of two-phase mixture in the remaining stages of the rotor may be described by Eqs. (4)-(9). The same problem arises when the mixer is used for the preliminary mixing of coarse mixtures, in the operation of dispersion apparatus for the production of finely disperse suspensions and emulsions, and in some rotor–film mass-transfer equipment.

The solution of Eqs. (4)-(9) with the boundary conditions

$$\delta = 0\ V_{1l} = V_{1\delta} = 0, \qquad \delta = \delta_1\ P = P_{atm},\ \partial V_{1l}/\partial\delta = 0 \tag{27}$$

takes the form

$$V_{1l} = \frac{\rho F_l}{\mu(\alpha_2)}\left(\delta_1\delta - \frac{\delta^2}{2}\right),\ V_{2l} = V_{1l} + \frac{\rho_2 F_l}{f(\alpha_2)}, \tag{28}$$

$$V_{1\delta}=(\sin\alpha F_l+RF_l')\frac{\rho}{6R\mu(\alpha_2)}\delta^3-[RF_l\delta_1'+(\sin\alpha F_l+RF_l')\delta_1]\frac{\rho\delta^2}{2R\mu(\alpha_2)}, \quad (29)$$

$$V_{2\delta}=V_{1\delta}+\frac{\alpha_1\alpha_2(\rho_2^0-\rho_1^0)}{f(\alpha_2)}F_\delta. \quad (30)$$

The unknown mixture thickness δ_1 is determined from the condition of constant flow rate, for example, of the liquid phase

$$\int_0^{\delta_1} 2\pi r\rho_1 V_{1l}d\delta=q_1, \quad (31)$$

$$\delta_1=\sqrt[3]{\left(\frac{q_1}{2\pi R\rho_1}\right)\left(\frac{3\mu(\alpha_2)}{\rho F_l}\right)}. \quad (32)$$

From the condition that δ_1 is of the order of the inclusion size, the generatrix length of the remaining stages of the rotor or other apparatus may be determined. Note that the solutions obtained are valid both for a monodisperse mixture and for particles of any size in a polydisperse mixture, since, if inertial terms are neglected in the equations of motion (the Stokes approximation), the equations describing the motion of the two-phase mixture in the two cases are the same, which follows from the results of [6]. Here $\alpha_2=\frac{4\pi}{3}\int a^3\varphi(a)da$, where $\Phi(a)$ is the size distribution function.

NOTATION

$\bar{V}_j$, ρ_j, α_j, velocity, mean density, and bulk concentration of the j-th phase of the mixture; ρ_j^0, true density of the j-th phase, $\bar{f}_{12}$, phase-interaction force; $f(\alpha_2)$, $\mu(\alpha_2)$, μ_1, phase-interaction force coefficient, effective viscosity of the mixture, viscosity of the carrier phase; q_j, mass flow rate of the j-th phase; ω, angular velocity of rotor rotation, P, pressure; a, particle size; ρ, mixture density.

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